

Probing New Physics with B_s mass-eigenstate asymmetries

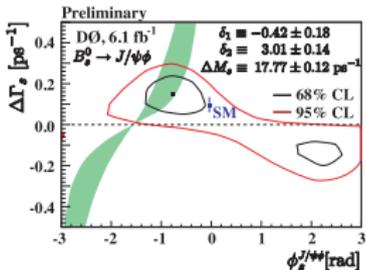


Rob Kneijens



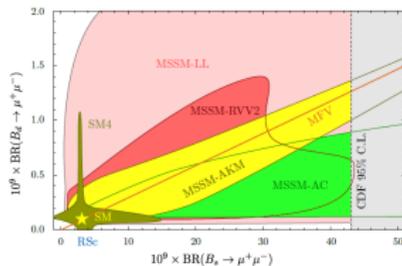
No smoking gun signal of New Physics from LHC

$$B_s \rightarrow J/\psi\phi$$



2010

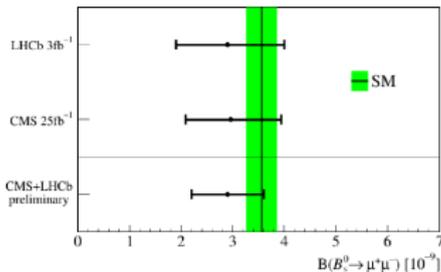
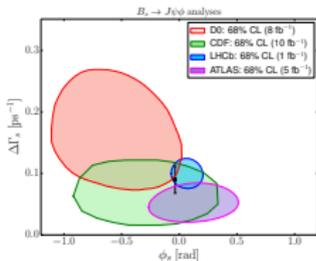
$$B_s \rightarrow \mu^+\mu^-$$



Straub; 1012.3893



2014



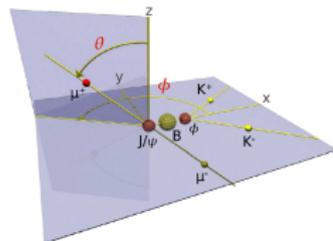
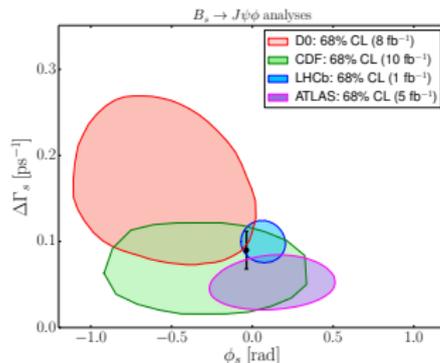
Smallish New Physics in B_s mixing?

LHCb $B_s \rightarrow J/\psi\phi$ analysis (1 fb^{-1}):

$$\phi_S = (4.0 \pm 5.2)^\circ$$

$$\Gamma_L - \Gamma_H = 0.100 \pm 0.017 \text{ ps}^{-1}$$

Compared to $\phi_S^{\text{SM}} = -(2.11 \pm 0.08)^\circ$

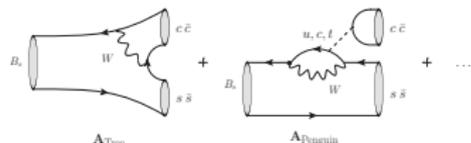


- Complicated experimental analysis
- Phase shift from penguin topologies?

Measure: $\phi_S + \Delta\phi_{J/\psi\phi}^\lambda$ for angular components λ

$$\Delta\phi_{J/\psi\phi}^\lambda = \mathcal{O}(1^\circ)$$

(See Faller, Fleischer, Mannel; 0810.4248)

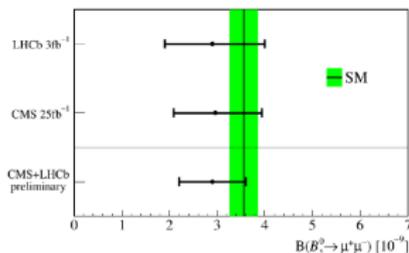


Complementary analyses desirable

Smallish New Physics in $B_s \rightarrow \mu^+ \mu^-$?

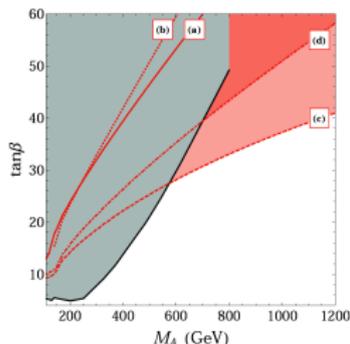
LHCb + CMS combined:

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-) = (2.9 \pm 0.7) \times 10^{-9}$$



Particularly sensitive to **scalar operators**:

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-) \propto m_\mu^2 \left[\left| (C_{10}^{\text{SM}} + C_{10}^{\text{NP}} - C'_{10}) + \frac{m_{B_s}}{2m_\mu} (C_P - C'_P) \right|^2 + \left| \frac{m_{B_s}}{2m_\mu} (C_S - C'_S) \right|^2 \right]$$



For example: MSSM with large $\tan \beta$:

$$C_S \simeq -C_P \approx \frac{m_\mu m_b}{8\pi \alpha} \frac{\tan^3 \beta}{M_A^2} \frac{\mu A_t}{M_{\tilde{t}_L}^2}$$

- Constructive interference with SM quite constraining
- Destructive interference with SM less so

Altmannshofer et al.; 1211.1976,

Complementary observables desirable

Complementary strategies to hunt for New Physics

- **Extracting CKM angle γ :**

$$B_s \rightarrow D_s^{\pm(*)} K^{\mp}$$

$$B_s \rightarrow K^+ K^-$$

with R.Fleischer, Nikhef LHCb group

- **Probing NP in $B_s^0-\bar{B}_s^0$ mixing:**

$$B_s \rightarrow K^+ K^-$$

$$B_s \rightarrow J/\psi f_0(980)$$

$$B_s \rightarrow J/\psi \eta^{(\prime)}$$

with R.Fleischer, G.Ricciardi

- **Identifying NP in $B_s \rightarrow \mu^+ \mu^-$**

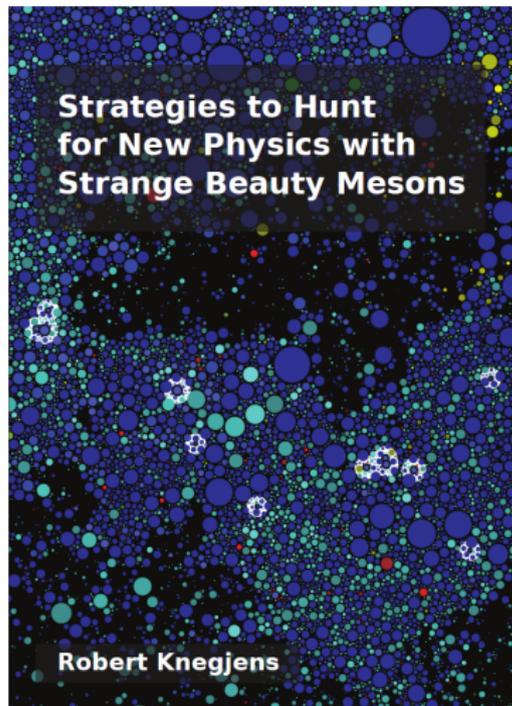
with R.Fleischer, A.Buras, J.Girrbach,

F.De Fazio, M.Nagai

Recurring observable:

$$\mathcal{A}_{\Delta\Gamma}^f = \frac{\Gamma(B_{s,H} \rightarrow f) - \Gamma(B_{s,L} \rightarrow f)}{\Gamma(B_{s,H} \rightarrow f) + \Gamma(B_{s,L} \rightarrow f)}$$

“mass-eigenstate rate asymmetries”

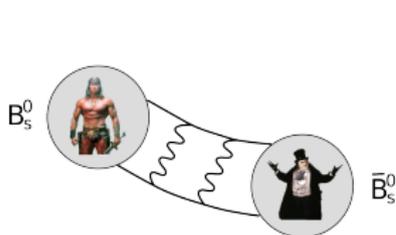


cover image: <http://paperscape.org>

B_s mass-eigenstates

Flavour basis B_s^0, \bar{B}_s^0 :

$$i \frac{d}{dt} \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix} = \begin{pmatrix} M_0 - \frac{i}{2}\Gamma_0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_0 - \frac{i}{2}\Gamma_0 \end{pmatrix} \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix}$$



"Normal"
Modes
→



Mass basis:

$$|B_{s,H,L}\rangle = p|B_s^0\rangle \pm q|\bar{B}_s^0\rangle$$

Well defined **masses** and
lifetimes

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} = -e^{-i\phi_s} \left[1 - \frac{1}{2} \underbrace{\text{Im} \left(\frac{\Gamma_{12}}{M_{12}} \right)}_{= a_{\text{SL}}^s \approx 0} + \mathcal{O} \left(\left| \frac{\Gamma_{12}}{M_{12}} \right|^2 \right) \right]$$

$$\phi_s \equiv \arg(M_{12}) \stackrel{\text{SM}}{=} 2\beta_s = -(2.1 \pm 0.1)^\circ$$

B_s decay width difference

$$K_S \approx K_+ \rightarrow \pi\pi$$

$$y_K = \frac{\Gamma_L - \Gamma_S}{\Gamma_L + \Gamma_S} \approx -1$$

analogously
 \implies

$$\mathcal{CP} |B_{s,\pm}\rangle = \pm |B_{s,\pm}\rangle$$

$$B_{s,+} \rightarrow D_s^{(*)} \bar{D}_s^{(*)}, \dots$$

$$y_s \equiv \frac{\Gamma_L - \Gamma_H}{\Gamma_L + \Gamma_H} = ?$$

In Standard Model expect $\phi_s \approx -2.1^\circ$:

$$\begin{pmatrix} |B_{s,L}\rangle \\ |B_{s,H}\rangle \end{pmatrix} = e^{-i\phi_s/2} \begin{pmatrix} \cos(\phi_s/2) & i \sin(\phi_s/2) \\ i \sin(\phi_s/2) & \cos(\phi_s/2) \end{pmatrix} \begin{pmatrix} |B_{s,+}\rangle \\ |B_{s,-}\rangle \end{pmatrix}$$

(convention dependent)



Estimate using Heavy Quark Expansion:

$$y_s|_{SM} = 0.067 \pm 0.016$$

A. Lenz, U. Nierste; 1102.4274

Indeed non-vanishing: LHCb collaboration; 1304.2600

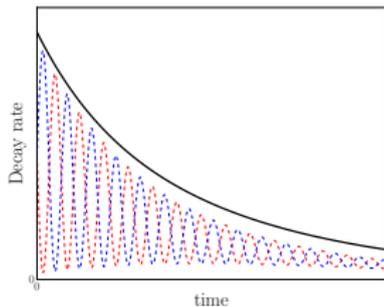
LHCb $B_s \rightarrow J/\psi \phi$ analysis :

$$y_s|_{LHCb} = 0.075 \pm 0.012$$

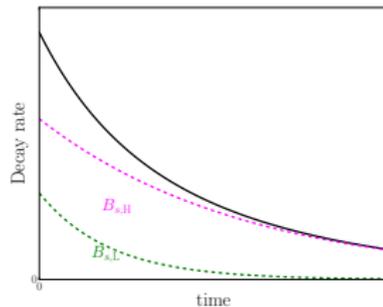
The untagged decay rate

$$\langle \Gamma(B_s(t) \rightarrow f) \rangle = \frac{1}{N_{B_s}} \frac{dN_e(B_s \rightarrow f)}{dt} = \dots$$

Flavour basis



Mass e-state basis



$$\Gamma(B_s^0(t) \rightarrow f) + \Gamma(\bar{B}_s^0(t) \rightarrow f)$$

$$\Gamma(B_{s,H} \rightarrow f) e^{-\Gamma_H t} + \Gamma(B_{s,L} \rightarrow f) e^{-\Gamma_L t}$$

$$\langle \Gamma_f \rangle = (\Gamma(B_{s,H} \rightarrow f) + \Gamma(B_{s,L} \rightarrow f)) e^{-t/\tau_{B_s}} \left\{ \cosh(y_s t/\tau_{B_s}) + \mathcal{A}_{\Delta\Gamma}^f \sinh(y_s t/\tau_{B_s}) \right\}$$

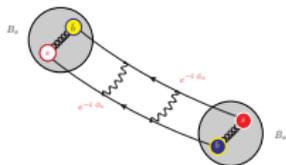
$$\mathcal{A}_{\Delta\Gamma}^f = \frac{\Gamma(B_{s,H} \rightarrow f) - \Gamma(B_{s,L} \rightarrow f)}{\Gamma(B_{s,H} \rightarrow f) + \Gamma(B_{s,L} \rightarrow f)}$$

$\mathcal{A}_{\Delta\Gamma}^f$: mass eigenstate rate asymmetry

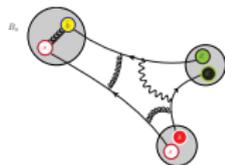
Consider: $B_s \rightarrow f$ with $\mathcal{CP}|f\rangle = \eta_f|f\rangle$

$$\lambda_f \equiv \frac{q}{p} \frac{A(\bar{B}_s^0 \rightarrow f)}{A(B_s^0 \rightarrow f)} = -\eta_f e^{-i\phi_s} \sqrt{\frac{1-C_f}{1+C_f}} e^{-i\Delta\phi_f}$$

$B_s^0 - \bar{B}_s^0$ **Mixing:**
 M_s, γ_s, ϕ_s



Decay Mode:
 $\Delta\phi_f, C_f$



Tagged CP observable: $\mathbf{S}_f = \frac{2 \text{Im} \lambda_f}{1 + |\lambda_f|^2} = \eta_f \sqrt{1 - C_f^2} \sin(\phi_s + \Delta\phi_f)$

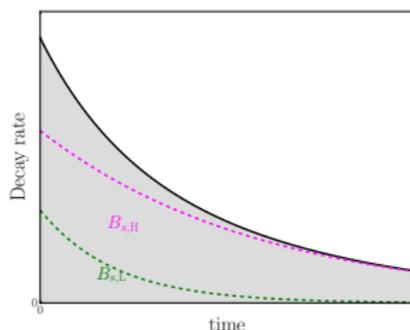
$$\mathcal{A}_{\Delta\Gamma}^f = \frac{2 \text{Re} \lambda_f}{1 + |\lambda_f|^2} = -\eta_f \sqrt{1 - C_f^2} \cos(\phi_s + \Delta\phi_f)$$

Probe New Physics with **untagged** time-dependent measurements?

Time-integrated untagged rate

Experiment measures:

$$\begin{aligned}\overline{\text{BR}}(B_s \rightarrow f) &\equiv \frac{1}{2} \int \langle \Gamma(B_s(t) \rightarrow f) \rangle dt \\ &= \frac{1}{2} \left[\frac{\Gamma(B_{s,H} \rightarrow f)}{\Gamma_H} + \frac{\Gamma(B_{s,L} \rightarrow f)}{\Gamma_L} \right]\end{aligned}$$



Theoretical calculation in flavor basis:

$$\Gamma(B_s^0 \rightarrow f) + \Gamma(\bar{B}_s^0 \rightarrow f) = \langle \Gamma(B_s(t) \rightarrow f) \rangle|_{t=0}$$

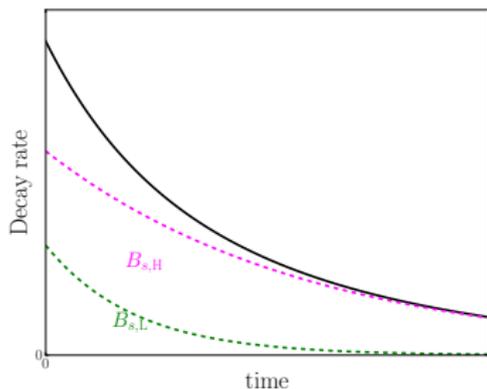
$$\text{BR}(B_s \rightarrow f) \equiv \frac{\langle \Gamma(B_s(t) \rightarrow f) \rangle|_{t=0}}{\frac{1}{2}(\Gamma_H + \Gamma_L)} = \frac{1}{2} \left[\frac{\Gamma(B_{s,H} \rightarrow f)}{\frac{1}{2}(\Gamma_H + \Gamma_L)} + \frac{\Gamma(B_{s,L} \rightarrow f)}{\frac{1}{2}(\Gamma_H + \Gamma_L)} \right]$$

Dictionary :
$$\overline{\text{BR}}(B_s \rightarrow f) = \text{BR}(B_s \rightarrow f) \left[\frac{1 + y_s \mathcal{A}_{\Delta\Gamma}^f}{1 - y_s^2} \right]$$

K. Bruyn, R. Fleischer, RK, P. Koppenburg, M. Merk, N. Tuning, Phys.Rev.D 86 (2012)

The Effective Lifetime

Approximate untagged rate $\langle \Gamma_f \rangle$ with single exponential $\frac{1}{\tau_f} e^{-t/\tau_f}$



Maximum likelihood fit gives:

$$\begin{aligned} \tau_f &= \frac{\int_0^\infty t \langle \Gamma_f \rangle dt}{\int_0^\infty \langle \Gamma_f \rangle dt} \\ &= \frac{\tau_{B_s}}{1 - y_s^2} \left(\frac{1 + 2 \mathcal{A}_{\Delta\Gamma}^f y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f y_s} \right) \end{aligned}$$

Recover theoretical BR:

$$\text{BR}(B_s \rightarrow f) = \underbrace{\left[2 - (1 - y_s^2) \frac{\tau_f}{\tau_{B_s}} \right]}_{\text{all measurable quantities}} \overline{\text{BR}}(B_s \rightarrow f)$$

Contours in the $\phi_s - \Delta\Gamma_s$ plane

$$\mathcal{CP}|f\rangle = \eta_f|f\rangle \quad \Rightarrow \quad \mathcal{A}_{\Delta\Gamma}^f = -\eta_f \sqrt{1 - C_f^2} \cos(\phi_s + \Delta\phi_f)$$

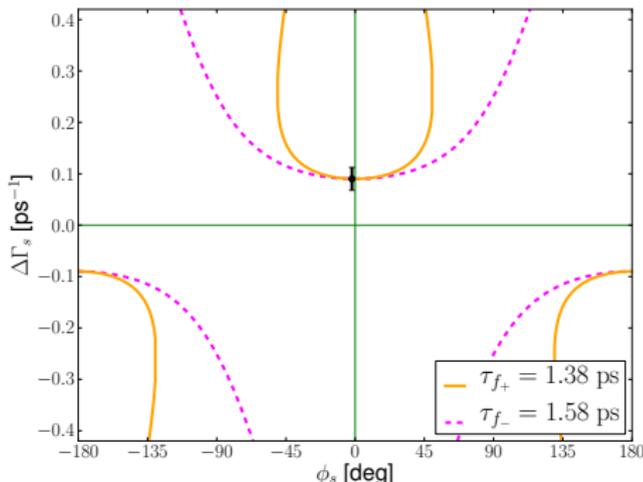
$$\tau_f = \frac{\tau_{B_s}}{1 - y_s^2} \left(\frac{1 + 2\mathcal{A}_{\Delta\Gamma}^f y_s + y_s^2}{1 + \mathcal{A}_{\Delta\Gamma}^f y_s} \right) = \text{function}(\Delta\Gamma_s, \phi_s + \Delta\phi_f, C_f)$$

Assuming:

$$\Delta\phi_f = 0, \quad C_f = 0$$

$$\mathcal{A}_{\Delta\Gamma}^f = \begin{cases} -\cos\phi_s & : f_{\text{even}} \\ +\cos\phi_s & : f_{\text{odd}} \end{cases}$$

(R. Fleischer, RK; 1109.5115)



Measured B_s effective lifetimes

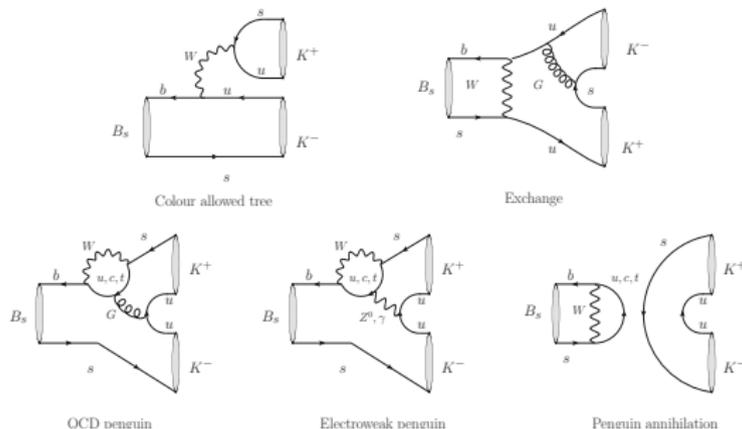
| Decay | CP e-val | Effective lifetime | |
|--------------------------------------|----------|--------------------------------|-----------------|
| $B_s \rightarrow K^+ K^-$ | even | $1.455 \pm 0.046 \pm 0.006$ ps | LHCb: 1207.5993 |
| $B_s \rightarrow D_s^+ D_s^-$ | even | $1.379 \pm 0.026 \pm 0.017$ ps | LHCb: 1312.1217 |
| $B_s \rightarrow J/\psi f_0(980)$ | odd | $1.700 \pm 0.040 \pm 0.026$ ps | LHCb: 1207.0878 |
| $B_s \rightarrow J/\psi \pi^+ \pi^-$ | odd | $1.652 \pm 0.024 \pm 0.024$ ps | LHCb: 1304.2600 |
| $B_s \rightarrow J/\psi K_S$ | odd | $1.75 \pm 0.12 \pm 0.07$ ps | LHCb: 1304.4500 |
| $B_s \rightarrow J/\psi \phi$ | mixed | $1.480 \pm 0.011 \pm 0.005$ ps | LHCb: 1402.2554 |
| $B_s \rightarrow D^- D_s^+$ | n/a | $1.52 \pm 0.15 \pm 0.01$ ps | LHCb: 1312.1217 |

But...

$$\Delta\phi \neq 0, C \neq 0$$

... possible CP violation in **Decay Modes**

A closer look at $B_s \rightarrow K^+ K^-$



$$A(B_s \rightarrow K^+ K^-) = \underbrace{V_{us} V_{ub}^*}_{\propto \lambda^4 e^{i\gamma}} (A_{tree}^u + A_{pen}^u - A_{pen}^t) + \underbrace{V_{cs} V_{cb}^*}_{\propto \lambda^2} (A_{pen}^c - A_{pen}^t)$$

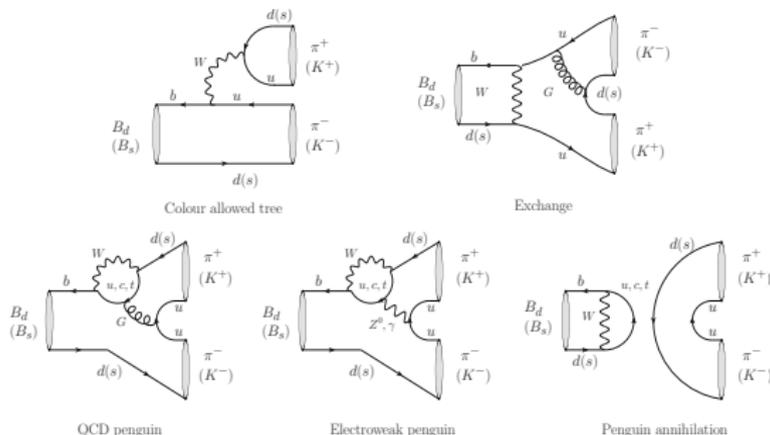
Direct and mixing-induced CPV: (LHCb: 1308.1428)

$$\mathcal{A}_{CP}^{\text{dir}}(B_s \rightarrow K^+ K^-) = 0.14 \pm 0.11 \pm 0.03$$

$$\mathcal{A}_{CP}^{\text{mix}}(B_s \rightarrow K^+ K^-) = -0.30 \pm 0.12 \pm 0.04$$

Controlling $B_s \rightarrow K^+ K^-$ with U -spin symmetry

U -spin symmetry: $s \leftrightarrow d$



1-1 topology mapping to $B_d \rightarrow \pi^+ \pi^-$ topologies:

$$\mathcal{A}_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-) = -0.29 \pm 0.05$$

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-) = 0.65 \pm 0.06$$

(HFAG average: Belle, BaBar, LHCb)

U -spin breaking: $f_K/f_\pi - 1$ or $(m_s - m_d)/\Lambda_{\text{QCD}} \implies \approx 20\%$

Extraction of γ

$$A(B_d \rightarrow \pi^+ \pi^-) = C [e^{i\gamma} - d e^{i\theta}] \quad A(B_s \rightarrow K^+ K^-) = \sqrt{\epsilon} C' \left[e^{i\gamma} + \frac{1}{\epsilon} d' e^{i\theta'} \right]$$

$$C^{(r)} \equiv A\lambda^3 R_b \left(A_{cc}^{u(r)} + A_{pen}^{u(r)} - A_{pen}^{t(r)} \right), \quad d^{(r)} e^{i\theta^{(r)}} \equiv \frac{1}{R_b} \frac{A_{pen}^{c(r)} - A_{pen}^{t(r)}}{A_{cc}^{u(r)} + A_{pen}^{u(r)} - A_{pen}^{t(r)}}.$$

$$\mathcal{A}_{CP}^{\text{dir,mix}}(B_d \rightarrow \pi^+ \pi^-)[d, \theta, \gamma, \phi_d] \xrightarrow{\text{fix } \phi_d} \text{contour in } \gamma\text{-}d \text{ plane}$$

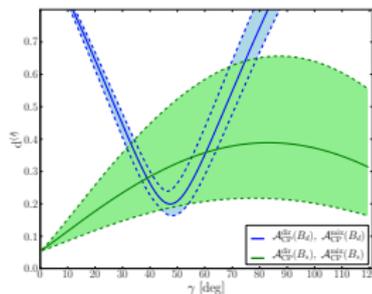
$$\mathcal{A}_{CP}^{\text{dir,mix}}(B_s \rightarrow K^+ K^-)[d', \theta', \gamma, \phi_s] \xrightarrow{\text{fix } \phi_s} \text{contour in } \gamma\text{-}d' \text{ plane}$$

Extract γ from U -spin assumption:

$$d = d'$$

(Fleischer, hep-ph/9903456)

Await more precise $B_s \rightarrow K^+ K^-$ CP asymmetries



Early determination of γ

$$K \equiv \frac{1}{\epsilon} \left| \frac{C}{C'} \right|^2 \left[\frac{M_{B_s}}{M_{B_d}} \frac{\Phi(M_\pi/M_{B_d}, M_\pi/M_{B_d})}{M_{B_d}} \frac{\tau_{B_d}}{\tau_{B_s}} \right] \frac{\text{BR}(B_s \rightarrow K^+ K^-)}{\text{BR}(B_d \rightarrow \pi^+ \pi^-)} = \frac{1}{\epsilon^2} \left[\frac{\epsilon^2 + 2\epsilon d' \cos \theta' \cos \gamma + d'^2}{1 - 2d \cos \theta \cos \gamma + d^2} \right]$$

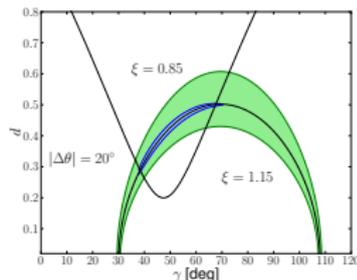
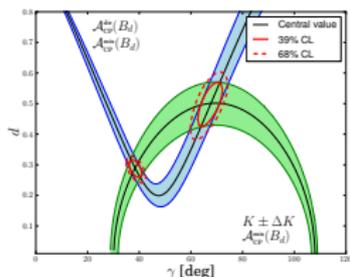
Factorisable U -spin breaking corrections:

$$\left| \frac{C'}{C} \right|_{\text{fact}} = \frac{f_K}{f_\pi} \frac{F_{B_s} K(M_K^2; 0^+)}{F_{B_d} \pi(M_\pi^2; 0^+)} \left(\frac{M_{B_s}^2 - M_K^2}{M_{B_d}^2 - M_\pi^2} \right) \stackrel{\text{QCDSR}}{=} 1.41^{+0.20}_{-0.11}$$

G. Duplancic, B. Melic; 0805.4170

Parameterise U -spin breaking: $\xi \equiv d/d' = 1 \pm 0.15$, $\Delta\theta \equiv \theta' - \theta = \pm 20^\circ$

(R. Fleischer, RK; 1011.1096)



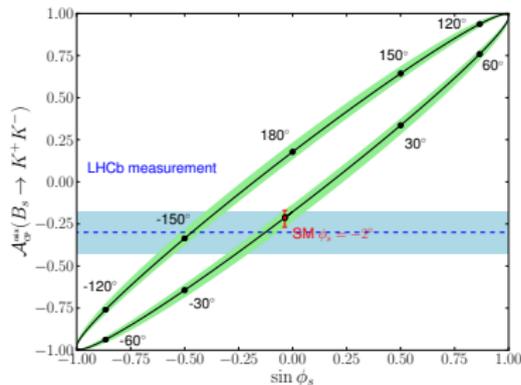
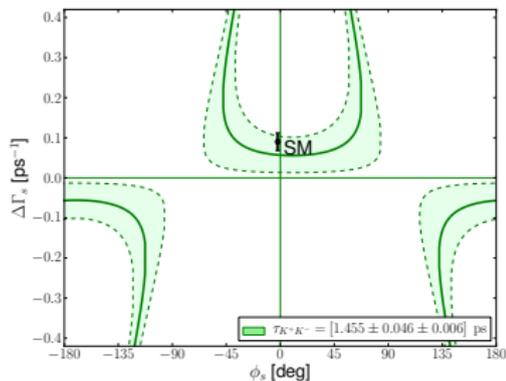
$$\gamma = (67.7^{+4.5}_{-5.0} |_{\text{input}} +5.0 |_{-3.7} |_{\xi} +0.1 |_{-0.2} |_{\Delta\theta})^\circ = (68 \pm 7)^\circ$$

Mixing-induced CPV in $B_s \rightarrow K^+ K^-$

$$\mathcal{A}_{\Delta\Gamma}(B_s \rightarrow K^+ K^-) = -\sqrt{1 - C_{K^+K^-}} \cos(\phi_s + \Delta\phi_{K^+K^-})$$

$$\mathcal{A}_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+ K^-) = \sqrt{1 - C_{K^+K^-}} \sin(\phi_s + \Delta\phi_{K^+K^-})$$

$$\Delta\phi_{K^+K^-} = 2\epsilon \sin\gamma \left[\frac{d \cos\theta + \epsilon \cos\gamma}{d^2 + 2\epsilon d \cos\theta \cos\gamma + \epsilon^2 \cos 2\gamma} \right] = - (10.7_{-2.1}^{+2.8})^\circ$$



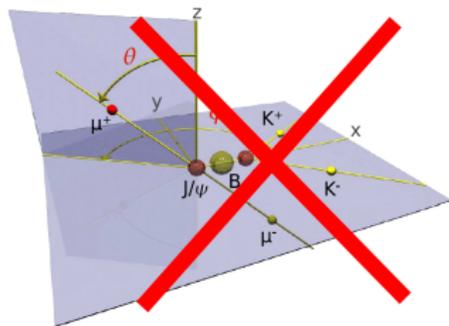
CP even lifetime contour

(R. Fleischer, RK; 1011.1096, 1109.5115)

A closer look at $B_s \rightarrow J/\psi f_0(980)$

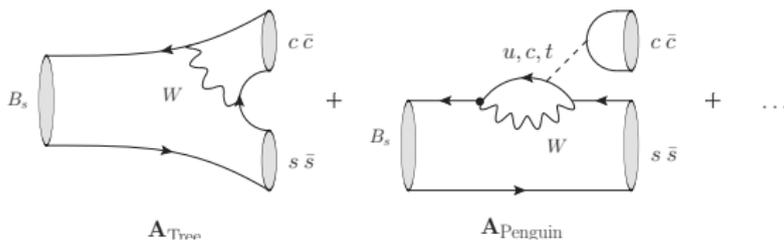
S.Stone, L.Zhang; 0812.2832

$$B_s \rightarrow J/\psi \cancel{\phi} \underbrace{f_0(980)}_{J^{PC}=0^{++}}$$

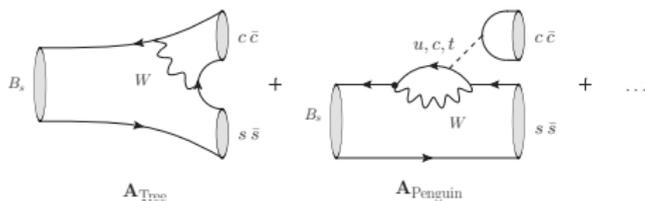


$$\frac{\overline{\text{BR}}(B_s \rightarrow J/\psi f_0(980); f_0 \rightarrow \pi^+ \pi^-)}{\overline{\text{BR}}(B_s \rightarrow J/\psi \phi; \phi \rightarrow K^+ K^-)} \approx \frac{1}{4}$$

Assuming $f_0(980) = s\bar{s} \rightarrow$ also $b \rightarrow c\bar{c}s$ transition:



$$B_s \rightarrow \bar{c}c\bar{s}s$$

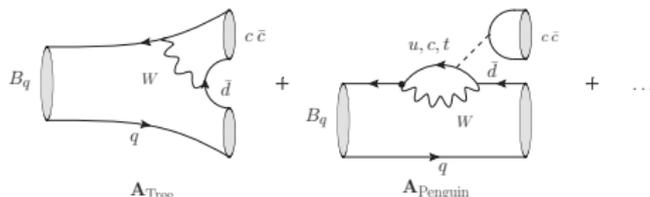


$$\begin{aligned}
 A(B_s \rightarrow \bar{c}c\bar{s}s) &= A_T V_{cb}^* V_{cs} + A_P^u V_{ub}^* V_{us} + A_P^c V_{cb}^* V_{cs} + A_P^t V_{tb}^* V_{ts} + \dots \\
 &= \mathcal{A} \left[1 + \underbrace{\epsilon}_{0.05} e^{i\gamma} b e^{i\theta} \right], \quad \left(\epsilon \equiv \frac{\lambda^2}{1-\lambda^2} \right)
 \end{aligned}$$

$$b e^{i\theta} = R_b \left(\frac{A_P^u - A_P^t}{A_T + A_P^c - A_P^t} \right)$$

$$\begin{cases} C &= -2\epsilon b \sin \gamma \sin \theta + \mathcal{O}(\epsilon^2) \\ \Delta\phi &= \underbrace{2\epsilon}_{6^\circ} b \sin \gamma \cos \theta + \mathcal{O}(\epsilon^2) \end{cases}$$

Penguin control via flavour symmetry



$$A(B_q \rightarrow \bar{c}c\bar{d}q) \stackrel{SU(3)_F}{=} -\lambda \mathcal{A} \left[1 - \underbrace{\kappa}_1 e^{i\gamma} b e^{i\theta} \right]$$

- Example: $B_d \rightarrow J/\psi K^0$ to $B_d \rightarrow J/\psi \pi^0$:

$$b \in [0.15, 0.67], \quad \theta \in [174^\circ, 212^\circ] \quad \implies \quad \Delta\phi_{J/\psi K^0}^d = [-3.9^\circ, -0.8^\circ]$$

S. Faller, R. Fleischer, M. Jung, T. Mannel, PRD 79 014030 (2009)

$$\Delta S_{B_d \rightarrow J/\psi K_s} = 0.00 \pm 0.02 \quad \implies \quad \Delta\phi_{J/\psi K^0}^d = (0.0 \pm 1.5)^\circ$$

M. Ciuchini, M. Pierini, L. Silvestrini, PRL 95 221804 (2005)/ 1102.0392

- Soon also $B_s \rightarrow J/\psi K^0$

K. De Bruyn, R. Fleischer, P. Koppenburg, Eur.Phys.J. C70 (2010) 1025-1035

$B_s \rightarrow J/\psi f_0(980)$ - What is the $f_0(980)$?

$f_0(980)$

[INSPIRE search](#)

See also the minireview on scalar mesons under $f_0(500)$. (See the index for the page number.)

$f_0(980)$ MASS

990 ± 20 MeV

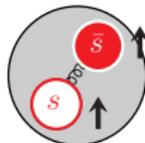
$f_0(980)$ WIDTH

40 to 1×10^2 MeV

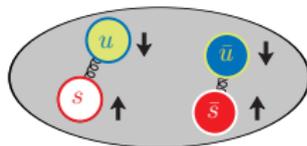
Decay Modes

| Γ_i | Mode | Fraction (Γ_i / Γ) | Scale Factor/ Confidence Level | p (MeV/c) |
|------------|-------------------------------------|----------------------------------|--------------------------------------|--------------|
| Γ_1 | $f_0(980) \rightarrow \pi\pi$ | dominant | | 476 |
| Γ_2 | $f_0(980) \rightarrow K\bar{K}$ | seen | | 36 |
| Γ_3 | $f_0(980) \rightarrow \gamma\gamma$ | seen | | 495 |
| Γ_4 | $f_0(980) \rightarrow e^+e^-$ | | | 495 |

Quark-antiquark



Tetraquark



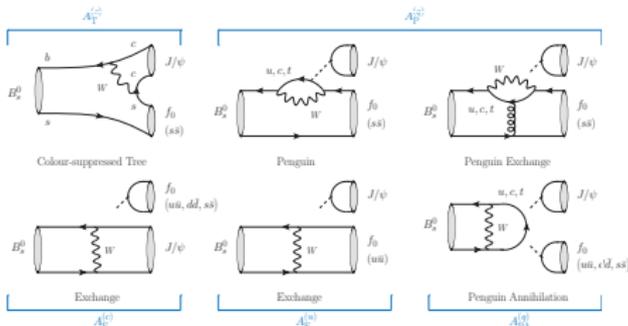
Or a $q\bar{q}$ – tetraquark mixture?

(see G.'t Hooft, G.Isidori, L.Maiani, A.Polosa; 0801.2288)

$f_0(980)$ as quark-antiquark state

$$\begin{pmatrix} |f_0(980)\rangle \\ |f_0(500)\rangle \end{pmatrix} = \begin{pmatrix} \cos \varphi_M & \sin \varphi_M \\ -\sin \varphi_M & \cos \varphi_M \end{pmatrix} \begin{pmatrix} |s\bar{s}\rangle \\ \frac{1}{\sqrt{2}} (|u\bar{u}\rangle + |d\bar{d}\rangle) \end{pmatrix}$$

Mixing phase φ_M largely unconstrained. R.Fleischer, RK, G.Ricciardi; 1109.1112

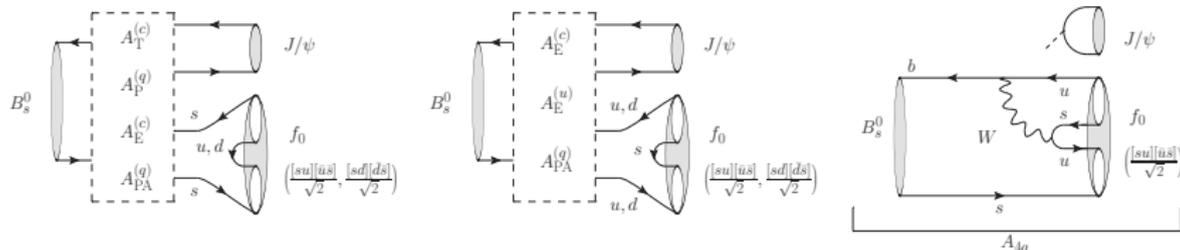


$$be^{i\vartheta} |_{q\bar{q}} = R_b \left[\frac{\cos \varphi_M \{ \tilde{A}_P^{(ut)} + \tilde{A}_{PA}^{(ut)} \} + \frac{1}{\sqrt{2}} \sin \varphi_M \{ \tilde{A}_E^{(u)} + 2\tilde{A}_{PA}^{(ut)} \}}{\cos \varphi_M \{ \tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + \tilde{A}_E^{(c)} + \tilde{A}_{PA}^{(ct)} \} + \frac{1}{\sqrt{2}} \sin \varphi_M \{ 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)} \}} \right]$$

$f_0(980)$ as a tetraquark

$$\begin{pmatrix} |f_0(980)\rangle \\ |f_0(500)\rangle \end{pmatrix} = \begin{pmatrix} \cos \omega & -\sin \omega \\ \sin \omega & \cos \omega \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} ([su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]) \\ [ud][\bar{u}\bar{d}] \end{pmatrix}$$

$$|\omega| < 5^\circ \quad (\text{see hep-ph/9808415, hep-ph/0407017, 0801.2288})$$



Ignoring mixing:

$$be^{i\vartheta} \Big|_{4q} = R_b \left[\frac{\tilde{A}_P^{(ut)} + \frac{1}{2}\tilde{A}_E^{(u)} + 2\tilde{A}_{PA}^{(ut)} + \frac{1}{2}\tilde{A}_{4q}}{\tilde{A}_T^{(c)} + \tilde{A}_P^{(ct)} + 2\tilde{A}_E^{(c)} + 2\tilde{A}_{PA}^{(ct)}} \right].$$

Proposed control channel: $B_d \rightarrow J/\psi f_0(980)$

Driven by $b \rightarrow c\bar{c}d \implies b'e^{i\theta'}$ not Cabibbo suppressed

- $q\bar{q}$: exact $SU(3)_F$ correspondence for $\varphi_M = \tan^{-1} \sqrt{2} = 55^\circ$
- in general require: $A_T, A_P \gg A_E, A_{PA}, (A_{4q})$

| Decay | Dominant topologies | BR bound |
|-----------------------------------|----------------------------|------------------------|
| $B_d \rightarrow J/\psi\phi$ | $A_E^{(c)}, A_{PA}^{(ct)}$ | $< 9.4 \times 10^{-7}$ |
| $B_s \rightarrow J/\psi\pi^0$ | $A_E^{(u)}$ | $< 1.2 \times 10^{-3}$ |
| $B_s \rightarrow J/\psi a^0(980)$ | $A_E^{(u)}, (A_{4q})$ | |

For example

$$\left| \frac{A_E^{(c)} + A_{PA}^{(ct)}}{A_T^{(c)}} \right| \sim \left(\frac{1 - \lambda^2/2}{\lambda} \right) \sqrt{\frac{\text{BR}(B_d \rightarrow J/\psi\phi)}{\text{BR}(B_d \rightarrow J/\psi K^{*0})}} \lesssim 0.1.$$

R.Fleischer, RK, G.Ricciardi; 1109.1112

Proposed control channel: $B_d \rightarrow J/\psi f_0(980)$

Assuming $\gamma = (68 \pm 7)^\circ$ and tree topologies dominant ($b \in [0, 0.5]$):

Predict : $\text{BR}(B_d \rightarrow J/\psi f_0; f_0 \rightarrow \pi^+ \pi^-)$

$$\sim (1 - 3) \times 10^{-6} \times \begin{cases} \left[\frac{\tan \varphi_M}{\tan 35^\circ} \right]^2 & : \quad q\bar{q} \\ 1 & : \quad \text{tetraquark} \end{cases}$$

R.Fleischer, RK, G.Ricciardi; 1109.1112

Experimental bound: (LHCb: 1301.5347)

$$\text{BR}(B_d \rightarrow J/\psi f_0(980); f_0 \rightarrow \pi^+ \pi^-) < 1.1 \times 10^{-6} \quad (90\% \text{ C.L.})$$

But also: (LHCb: 1402.6248)

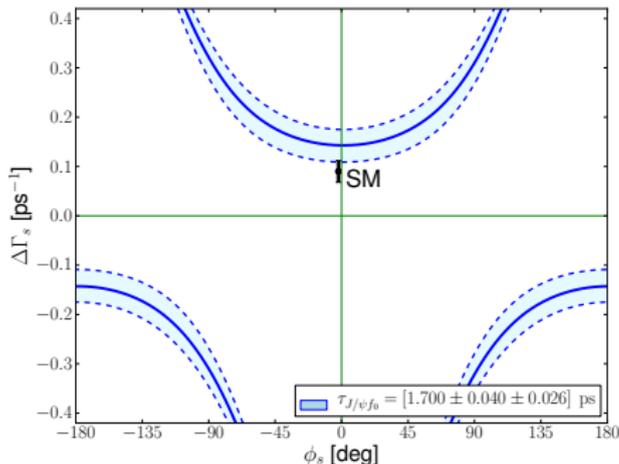
$$\frac{\text{BR}(B_s \rightarrow J/\psi f_0(500); f_0 \rightarrow \pi^+ \pi^-)}{\text{BR}(B_s \rightarrow J/\psi f_0(980); f_0 \rightarrow \pi^+ \pi^-)} < 3.4\% \quad (95\% \text{ C.L.})$$

$$\implies \varphi_M < 7.7^\circ$$

$B_s \rightarrow J/\psi f_0(980)$ effective lifetime (CP-odd)

Assuming $\gamma = (68 \pm 7)^\circ$ and tree topologies dominant ($b < 0.5$):

$$\Delta\phi_{J/\psi f_0} \in [-3^\circ, 3^\circ] \quad C_{J/\psi f_0} \lesssim 0.05$$



CP odd lifetime contour

A quick look at $B_s \rightarrow J/\psi\eta^{(\prime)}$

$$|\eta\rangle = \cos\phi_P \frac{1}{\sqrt{2}} (|u\bar{u}\rangle + |d\bar{d}\rangle) - \sin\phi_P |s\bar{s}\rangle$$

$$|\eta'\rangle = \cos\phi_G \sin\phi_P \frac{1}{\sqrt{2}} (|u\bar{u}\rangle + |d\bar{d}\rangle) + \cos\phi_G \cos\phi_P |s\bar{s}\rangle + \sin\phi_G |gg\rangle$$

$$30^\circ \lesssim \phi_P \lesssim 45^\circ$$

$$|\phi_G| \sim 20^\circ$$

C. Di Donato, G. Ricciardi, I. Bigi, Phys.Rev. D85 013016 (2012)

A.S. Dighe, M. Gronau, J.L.Rosner, Phys.Lett. B367 (1996) 357-361

Difficult LHC signature: $\eta^{(\prime)} \xrightarrow{\text{prominently}} \gamma, \pi^0$

Belle results:

$$\text{BR}(B_s \rightarrow J/\psi\eta) = 5.10_{-0.97}^{+1.30} \times 10^{-4} \quad \text{BR}(B_d \rightarrow J/\psi\eta) = 12.3_{-1.8}^{+1.9} \times 10^{-6}$$

$$\text{BR}(B_s \rightarrow J/\psi\eta') = 3.71_{-0.85}^{+1.00} \times 10^{-4} \quad \text{BR}(B_d \rightarrow J/\psi\eta') < 7.4 \times 10^{-6}$$

M. C. Chang et al, Phys. Rev. D85 (2012) 091102, Belle Collaboration, Phys. Rev. Lett. 108 (2012) 181808

Determining the $\eta^{(\prime)}$ mixing angles

Assuming $A_T, A_P \gg A_E, A_{PA}$:

$$R_s \equiv \frac{\text{BR}(B_s \rightarrow J/\psi\eta')}{\text{BR}(B_s \rightarrow J/\psi\eta)} \left(\frac{\Phi_s^\eta}{\Phi_s^{\eta'}} \right)^3$$

$$= \frac{\cos^2 \phi_G}{\tan^2 \phi_P} = 0.91 \pm 0.18$$

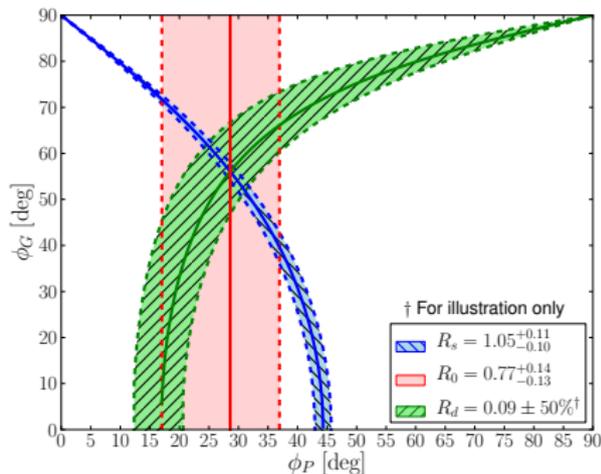
$$R_0 \equiv \frac{\text{BR}(B_d \rightarrow J/\psi\eta)}{\text{BR}(B_d \rightarrow J/\psi\pi^0)} \left(\frac{\Phi_d^\pi}{\Phi_d^\eta} \right)^3$$

$$= \cos^2 \phi_P = 0.77 \pm 0.14$$

$$R_d \equiv \frac{\text{BR}(B_d \rightarrow J/\psi\eta')}{\text{BR}(B_d \rightarrow J/\psi\eta)} \left(\frac{\Phi_d^\eta}{\Phi_d^{\eta'}} \right)^3$$

$$= \cos^2 \phi_G \tan^2 \phi_P = ??$$

Φ_q^P : phase space

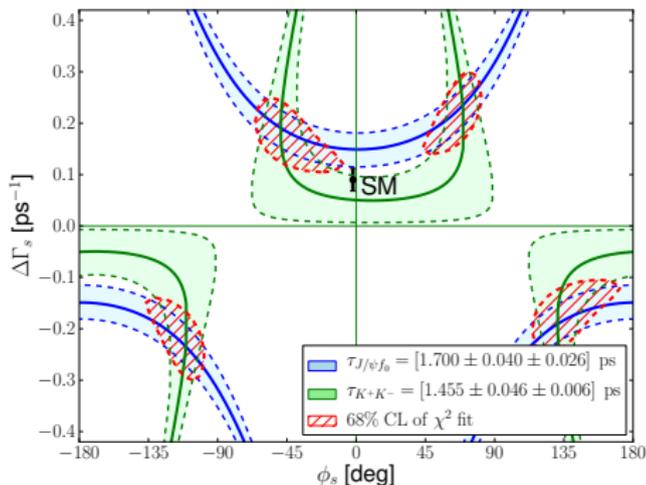


R.Fleischer, RK, G.Ricciardi; 1110.5490

Lifetime contours in the $\phi_s - \Delta\Gamma_s$ plane

$$\tau_f = \text{function} \left(\Delta\Gamma_s, \boxed{\phi_s + \Delta\phi}, C \right)$$

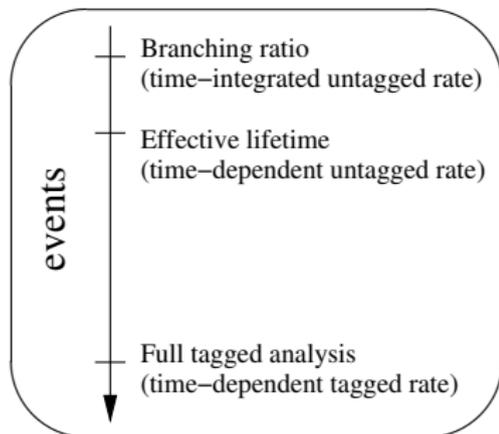
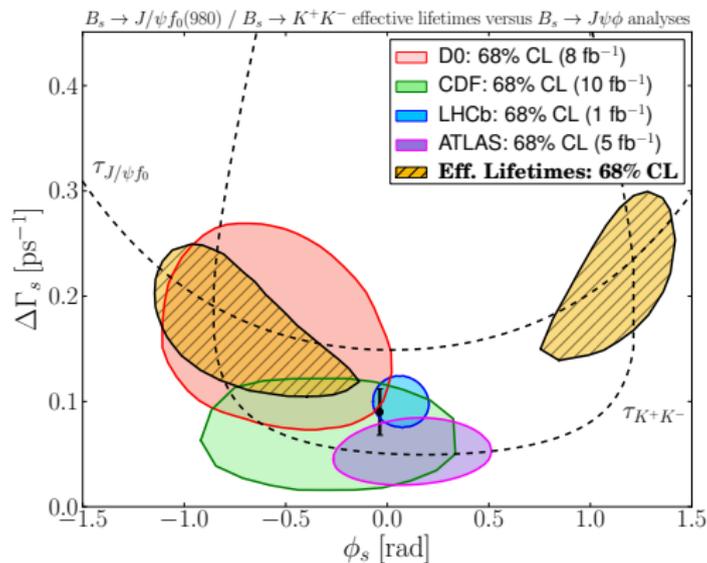
- **CP Even** : τ_{K+K^-} , $\Delta\phi_{K+K^-} = - (10.5^{+3.1}_{-2.8})^\circ$, $C_{K+K^-} = 0.09$
- **CP Odd** : $\tau_{J/\psi f_0}$, $\Delta\phi_{J/\psi f_0} \in [-3^\circ, 3^\circ]$, $C_{J/\psi f_0} \leq 0.05$



R. Fleischer and RK, *Eur.Phys.J. C71* (2011) 1532, RK, C12-06-11.2; 1209.3206

Comparison with tagged measurements

Full tagged $B_s \rightarrow J/\psi\phi$ analysis:



Upcoming *untagged time-dependent* measurements?

The Rare Decay $B_s \rightarrow \mu^+ \mu^-$

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-) = \begin{cases} (2.9^{+1.1}_{-1.0}) \times 10^{-9} & \text{LHCb} \\ (3.0^{+1.0}_{-0.9}) \times 10^{-9} & \text{CMS} \end{cases} = (2.9 \pm 0.7) \times 10^{-9}$$

LHCb: *Phys.Rev.Lett.* 111 (2013) 101805, CMS: *Phys.Rev.Lett.* 111 (2013) 101804

To be compared with:

$$\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9}$$

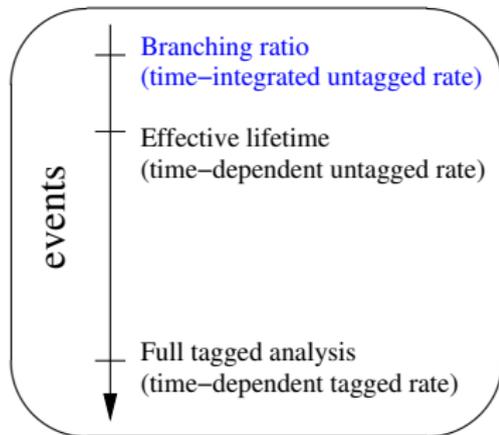
Includes:

- NLO EW and NNLO QCD effects

C. Bobeth, M. Gorbahn, T. Hermann, M. Misiak, E. Stamou, M. Steinhauser; Phys.Rev.Lett. 112 (2014) 101801

- y_s effect ($\mathcal{A}_{\Delta\Gamma}^{\mu\mu}|_{\text{SM}} = 1$)

K. Bruyn, R. Fleischer, RK, P. Koppenburg, M. Merk, A. Pellegrino, N. Tuning; Phys.Rev.Lett 109 (2012)



$$\bar{R} \equiv \frac{\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)}{\overline{\text{BR}}(B_s \rightarrow \mu^+ \mu^-)_{\text{SM}}},$$

$$\bar{R}_{\text{LHCb}} = 0.79 \pm 0.20, \quad \bar{R}_{\text{SM}} = 1$$

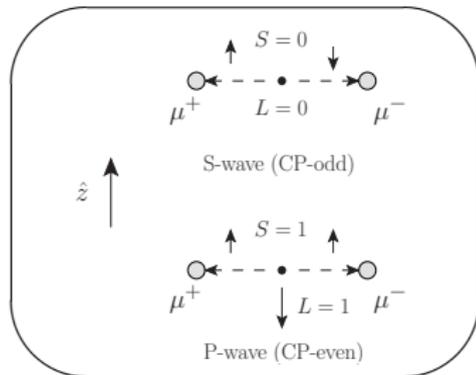
$B_s \rightarrow \mu^+ \mu^-$ beyond the Standard Model

$$\mathcal{H}_{\text{eff}} = -\frac{G_F \alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \sum_i^{\{10, S, P\}} (C_i \mathcal{O}_i + C'_i \mathcal{O}'_i)$$

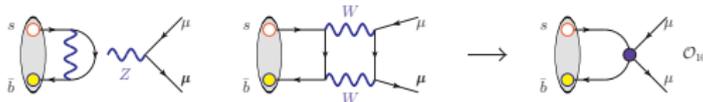
$$\mathcal{O}_{10} = (\bar{s} \gamma_\mu P_L b) (\bar{\mu} \gamma^\mu \gamma_5 \mu)$$

$$\mathcal{O}_S = (\bar{s} P_R b) (\bar{\mu} \mu)$$

$$\mathcal{O}_P = (\bar{s} P_R b) (\bar{\mu} \gamma_5 \mu) \quad (P_L \leftrightarrow P_R \text{ for } \mathcal{O}')$$



Standard Model: only $\mathcal{O}_{10} \implies$ only $B_{s,H} \rightarrow \mu^+ \mu^-$



Beyond the SM: non-vanishing $B_{s,L} \rightarrow \mu^+ \mu^-$?



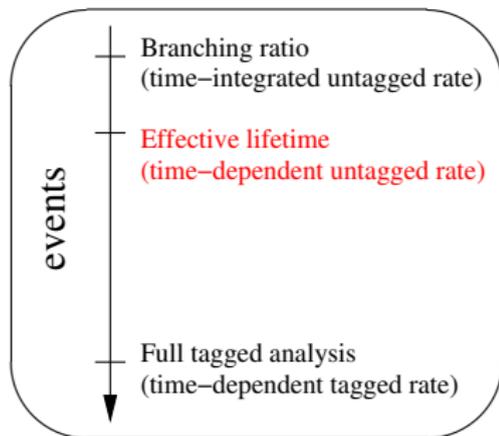
$B_s \rightarrow \mu^+ \mu^-$ time-dependent measurement

Define for convenience:

$$\mathbf{P} \equiv \frac{C_{10} - C'_{10}}{C_{10}^{\text{SM}}} + \frac{m_{B_s}}{2 m_\mu} \left(\frac{C_P - C'_P}{C_{10}^{\text{SM}}} \right)$$

$$\mathbf{S} \equiv \sqrt{1 - \frac{4 m_\mu^2}{m_{B_s}^2}} \frac{m_{B_s}}{2 m_\mu} \left(\frac{C_S - C'_S}{C_{10}^{\text{SM}}} \right)$$

In SM: $\mathbf{P} \rightarrow 1$, $\mathbf{S} \rightarrow 0$

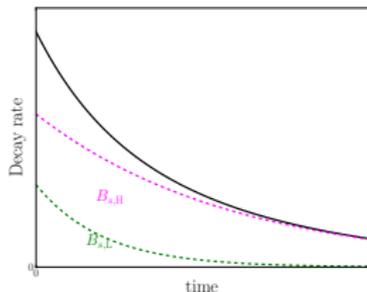


$$\Gamma(B_{s,L} \rightarrow \mu^+ \mu^-) \propto |\mathbf{P}|^2 \underbrace{\sin^2(\varphi_P - \phi_s^{\text{NP}}/2)}_{\text{new CP phases}} + |\mathbf{S}|^2 \underbrace{\cos^2(\varphi_S - \phi_s^{\text{NP}}/2)}_{\text{scalar operators}}$$

Probe **NP** with : $\mathcal{A}_{\Delta\Gamma}^{\mu\mu} \equiv \frac{\Gamma(B_{s,H} \rightarrow \mu^+ \mu^-) - \Gamma(B_{s,L} \rightarrow \mu^+ \mu^-)}{\Gamma(B_{s,H} \rightarrow \mu^+ \mu^-) + \Gamma(B_{s,L} \rightarrow \mu^+ \mu^-)}$

e.g. with $B_s \rightarrow \mu^+ \mu^-$ **Effective Lifetime**

$B_s \rightarrow \mu^+ \mu^-$ untagged observables



$$\bar{R} = \frac{(1 + y_s \mathcal{A}_{\Delta\Gamma}^{\mu\mu})}{1 + y_s} (|P|^2 + |S|^2)$$

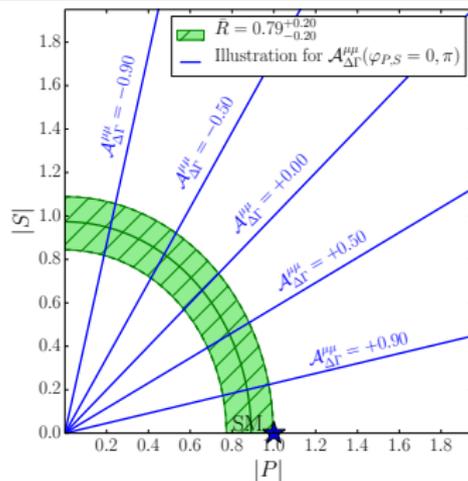
$$\mathcal{A}_{\Delta\Gamma}^{\mu\mu} = \frac{|P|^2 \cos(2\varphi_P - \phi_s^{\text{NP}}) - |S|^2 \cos(2\varphi_S - \phi_s^{\text{NP}})}{|P|^2 + |S|^2}$$

K. Bruyn, R. Fleischer, RK, P. Koppenburg, M. Merk, A. Pellegrino, N. Tuning, Phys.Rev.Lett 109 (2012)

Solvable scenarios:

- A: $|P|$, φ_P free ($S = 0$)
- B: $|S|$, φ_S free ($P = 1$)
- C: $S = \pm[1 - P]$
- D: $\varphi_P = \varphi_S = 0$: \rightarrow

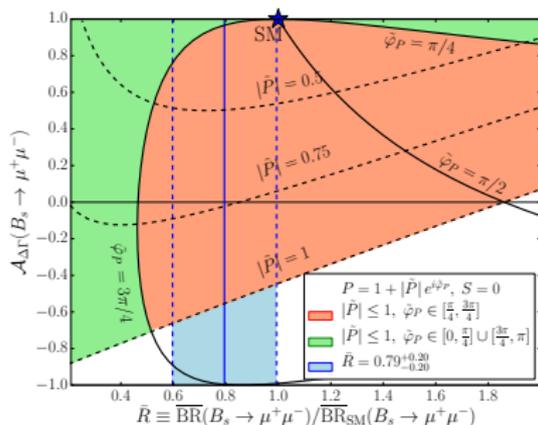
A.J. Buras, R. Fleischer, J. Girrbach, RK, JHEP 1307 (2013) 77



No scalar operators || only new scalar operators

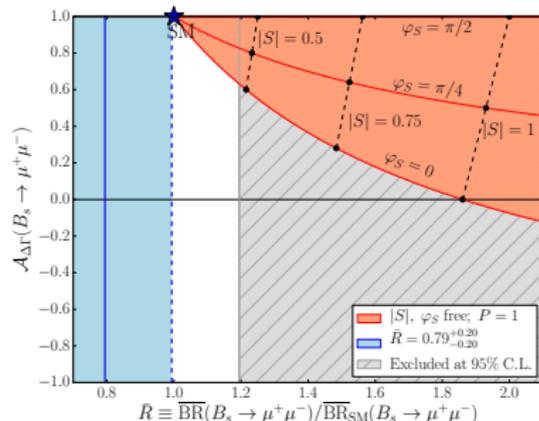
$$\Gamma(B_{s,L} \rightarrow \mu^+ \mu^-) \propto \underbrace{|P|^2 \sin^2(\varphi_P - \phi_s^{\text{NP}}/2)}_{\text{scenario A}} + \underbrace{|S|^2 \cos^2(\varphi_S - \phi_s^{\text{NP}}/2)}_{\text{scenario B}}$$

Scenario A: $S = 0$



E.g: CMFV, Z' Models,
 A^0 dominant (2HDM)

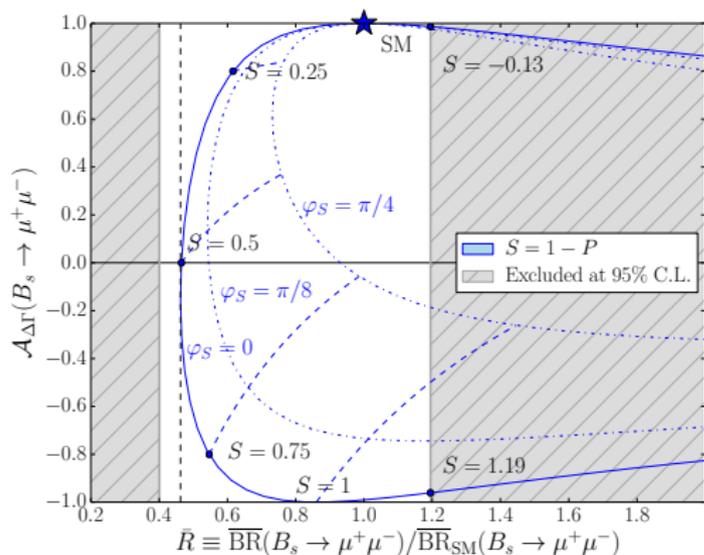
Scenario B: $S \neq 0$ ($P = 1$)



E.g: H^0 dominant (2HDM)

New scalar and pseudoscalar operators on same footing

Scenario C: $P = 1 + \tilde{P}$, $S = \pm \tilde{P}$



Realised for:

$$C_S^{(\prime)} = \pm C_P^{(\prime)}$$

Example

2HDM in decoupling regime:

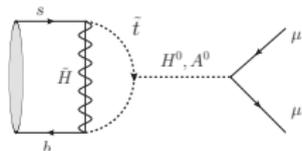
$$M_{H^0} \simeq M_{A^0} \simeq M_{H^\pm} \gg M_{h^0}$$

$$C_S \simeq -C_P$$

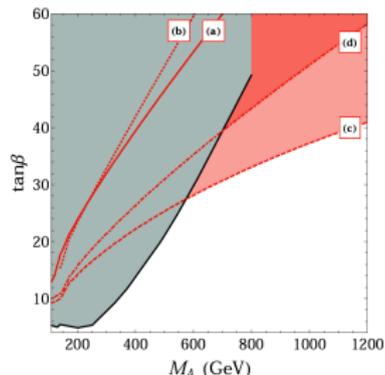
$$\left(\simeq \frac{m_b}{m_s} C_S' \simeq \frac{m_b}{m_s} C_P' \right) \text{ (MFV)}$$

- Full range of $\mathcal{A}_{\Delta\Gamma}^{\mu\mu}$ without CP violating phases
- Lower bound $\bar{R} \geq (1 - y_s)/2$

The MSSM with large $\tan\beta$



$$C_S \simeq -C_P \simeq \frac{m_\mu m_b \tan^3 \beta}{8\pi \alpha} \frac{\mu A_t}{M_A^2} \frac{\mu A_t}{M_{\tilde{t}_L}^2} \quad (\text{MFV})$$

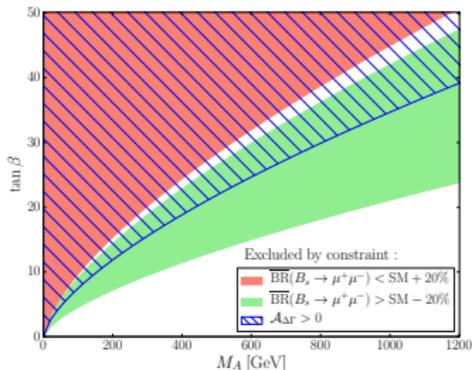
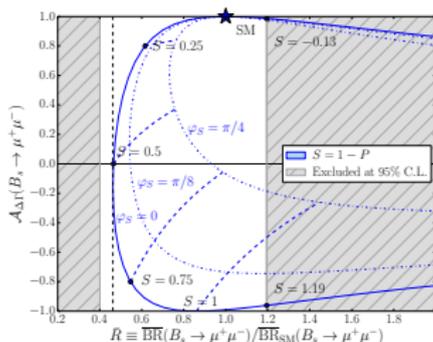


Altmannshofer et al.; 1211.1976, 1306.0022

(a) $\mu = 1 \text{ TeV}$, $A_t > 0$ (destructive interference)

(c) $\mu = -1.5 \text{ TeV}$, $A_t > 0$ (constructive interference)

$\tilde{M}_q = 2 \text{ TeV}$, $|A_t|$ tuned s.t. $M_h = 126 \text{ GeV}$



Example of destructive interference scenario

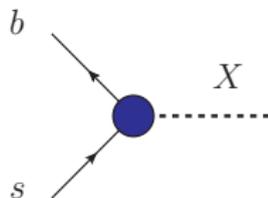
Loose bound on $\mathcal{A}_{\Delta\Gamma}$ can rule out “large $\tan\beta$ ” allowed regions

Compatibility with B_s mixing constraints

Consider generic models:

$$X \in \{ \mathbf{Z}', \mathbf{H}^0, \mathbf{A}^0, \mathbf{H}^0 + \mathbf{A}^0 \},$$

$$M_X = 1 \text{ TeV}$$



$$\mathcal{L}_{\text{FCNC}}(\mathbf{Z}') = [\Delta_L^{sb}(\mathbf{Z}') \bar{s} \gamma_\mu P_L b + \Delta_R^{sb}(\mathbf{Z}') \bar{s} \gamma_\mu P_R b] \mathbf{Z}'^\mu$$

$$\mathcal{L}_{\text{FCNC}}(\mathbf{H}) = [\Delta_L^{sb}(\mathbf{H}) \bar{s} P_L b + \Delta_R^{sb}(\mathbf{H}) \bar{s} P_R b] \mathbf{H}$$

A.J. Buras, F. De Fazio, J. Girrbach, JHEP 1302 (2013) 116

A.J. Buras, F. De Fazio, J. Girrbach, RK, M. Nagai, JHEP 1306 (2013) 111

Including $\Delta F = 2$ NLO corrections: *A.J. Buras, J. Girrbach, JHEP 1203 (2012) 052*

- Apply B_s mixing constraints:

$$\Delta M_s \in \Delta M_{s,\text{exp}}^{\text{cent. val.}} \pm 5\%, \quad \phi_s \in \phi_{s,\text{exp}}^{\text{cent. val.}} \pm 2\sigma$$

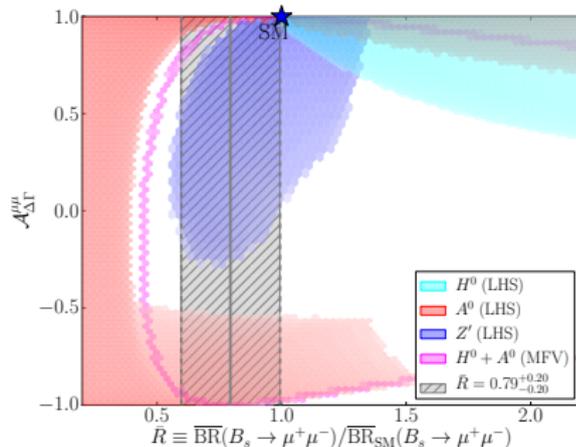
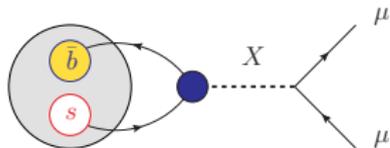
Specific models in the $\overline{R}-\mathcal{A}_{\Delta\Gamma}^{\mu\mu}$ parameter space

$$X \in \{ Z', H^0, A^0, H^0 + A^0 \},$$

$$M_X = 1 \text{ TeV}$$

- B_s mixing ($\Delta M_s, \phi_s$)
constrains quark couplings
- Lepton couplings left free
(no CPV)
- Z' includes combined
 $b \rightarrow sll$ constraints on C_{10}

W.Altmannshofer, D.Straub; 1206.0273



A.J. Buras, R. Fleischer, J. Girrbach, RK, JHEP 1307 (2013) 77

A.J. Buras, F. De Fazio, J. Girrbach, RK, M. Nagai, JHEP 1306 (2013) 111

$B_s \rightarrow \mu^+ \mu^-$ tagged analysis

Eventually also tagged measurement:

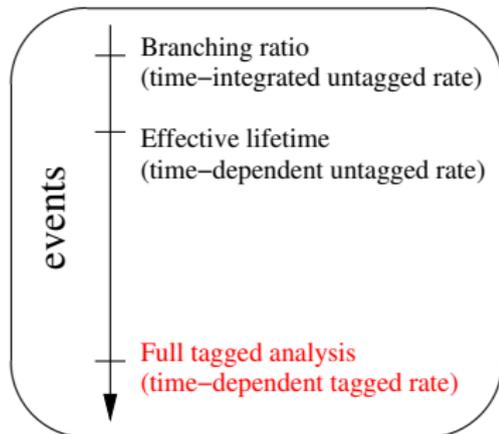
$$\frac{\Gamma(B_s^0(t) \rightarrow \mu^+ \mu^-) - \Gamma(\bar{B}_s^0(t) \rightarrow \mu^+ \mu^-)}{\Gamma(B_s^0(t) \rightarrow \mu^+ \mu^-) + \Gamma(\bar{B}_s^0(t) \rightarrow \mu^+ \mu^-)}$$
$$= \frac{\mathcal{S}_{\mu\mu} \sin(\Delta M_s t)}{\cosh(y_s t / \tau_{B_s}) + \mathcal{A}_{\Delta\Gamma}^{\mu\mu} \sinh(y_s t / \tau_{B_s})}$$

- $\mathcal{S}_{\mu\mu}$ **independent** if scalar operators:

$$|\mathcal{A}_{\Delta\Gamma}^{\mu\mu}|^2 + |\mathcal{S}_{\mu\mu}|^2$$
$$= 1 - \left[\frac{2|P||S| \cos(\varphi_P - \varphi_S)}{|P|^2 + |S|^2} \right]^2$$

- $\mathcal{S}_{\mu\mu}$ sensitive to small CP phases:

$$\mathcal{S}_{\mu\mu} = \frac{|P|^2 \sin(2\varphi_P - \phi_s^{\text{NP}}) - |S|^2 \sin(2\varphi_S - \phi_s^{\text{NP}})}{|P|^2 + |S|^2}$$



Summary

- No **smoking gun signal of NP** \implies smallish NP? **complementary strategies** to flagship analyses desirable!
- Sizable B_s width difference ($y_s \approx 7\%$) allows extraction of **mass-eigenstate rate asymmetries** ($\mathcal{A}_{\Delta\Gamma}^f$) from time-dependent untagged measurements
e.g. **effective lifetimes**

important ingredient for: $\text{BR}(B_s \rightarrow f) \xleftrightarrow{\mathcal{A}_{\Delta\Gamma}^f/\tau_f} \overline{\text{BR}}(B_s \rightarrow f)$

- Probe $B_s^0 - \bar{B}_s^0$ mixing parameters from pair of effective lifetimes (τ_{f+}, τ_{f-}),
- Effective lifetime of $B_s \rightarrow \mu^+ \mu^-$ complements branching ratio for searching for NP (particularly for new scalars!)



Backup slides

Fitting an effective lifetime

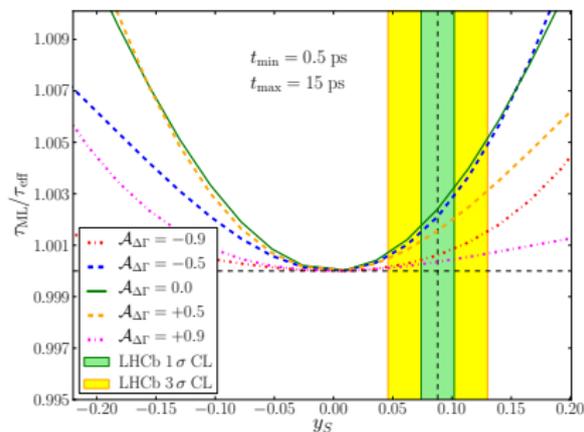
$$f_{\text{true}}(t) \equiv \frac{A(t) \langle \Gamma(t) \rangle}{\int_0^\infty A(t) \langle \Gamma(t) \rangle dt}, \quad f_{\text{fit}}(t; \tau) \equiv \frac{A(t) e^{-t/\tau}}{\int_0^\infty A(t) e^{-t/\tau} dt}$$

Minimise : $-\log L(\tau) = -n \int_0^\infty dt f_{\text{true}}(t) \log [f_{\text{fit}}(t; \tau)]$

$$\frac{\int_0^\infty t A(t) e^{-t/\tau} dt}{\int_0^\infty A(t) e^{-t/\tau} dt} = \frac{\int_0^\infty t A(t) \langle \Gamma(t) \rangle dt}{\int_0^\infty A(t) \langle \Gamma(t) \rangle dt}$$

Limit that $A(t) = 1$:

$$\tau = \frac{\int_0^\infty t \langle \Gamma(t) \rangle dt}{\int_0^\infty \langle \Gamma(t) \rangle dt} \equiv \tau_{\text{eff}},$$



Tetraquarks

- diquark–antidiquark (colour) bound states

$$\sigma = [ud][\bar{u}\bar{d}]$$

$$\kappa = [su][\bar{u}\bar{d}]; [sd][\bar{u}\bar{d}] \quad (+c.d)$$

$$f_0 = \frac{[su][\bar{s}\bar{u}] + [sd][\bar{s}\bar{d}]}{\sqrt{2}}$$

$$a_0 = [su][\bar{s}\bar{d}]; \frac{[su][\bar{s}\bar{u}] - [sd][\bar{s}\bar{d}]}{\sqrt{2}}; [sd][\bar{s}\bar{u}]$$

diquark $\equiv [q_1 q_2]$, colour $\bar{\mathbf{3}}$, flavour $\bar{\mathbf{3}}$, $S = 0$

- Issues: $f_0 \rightarrow \pi\pi$ coupling too small, $a_0 \rightarrow \eta\pi$ too large.
- Solved by adding *instanton-induced effects*

A Theory of Scalar Mesons, G. 't Hooft, G. Isidori, A.D Polosa, V. Riquer,

(arXiv:0801.2288)

Tagged analysis

The CP asymmetry:

$$\frac{\Gamma(B_s(t) \rightarrow f) - \Gamma(\bar{B}_s(t) \rightarrow f)}{\Gamma(B_s(t) \rightarrow f) + \Gamma(\bar{B}_s(t) \rightarrow f)} = \frac{C \cos(\Delta M_s t) + S \sin(\Delta M_s t)}{\cosh(\Delta \Gamma_s t) + \mathcal{A}_{\Delta \Gamma} \sinh(\Delta \Gamma_s t)}$$

Observables for $\mathcal{CP}|f\rangle = \eta|f\rangle$:

$$\lambda_f \equiv \frac{q}{p} \frac{A(\bar{B}_s^0 \rightarrow f)}{A(B_s^0 \rightarrow f)} = -\eta e^{-i\phi_s} \sqrt{\frac{1-C}{1+C}} e^{-i\Delta\phi}$$

$$\mathcal{A}_{\Delta\Gamma} + iS = \frac{2\lambda_f}{1 + |\lambda_f|^2} = \boxed{-\eta \sqrt{1 - C^2} e^{-i(\phi_s + \Delta\phi)}}$$